

# Optimization methods for the memory allocation problems in embedded systems

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# Outline

- 1 Introduction
  - Embedded systems
  - Memory allocation
  - Conflict graph
- 2 Memory allocation problems
  - Unconstrained memory allocation problem
  - Allocation with constraint on the number of memory banks
  - General memory allocation problem
  - Dynamic memory allocation problem
- 3 Conclusions and future work

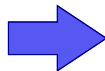


# Outline

- 1 Introduction
  - Embedded systems
  - Memory allocation
  - Conflict graph
- 2 Memory allocation problems
- 3 Conclusions and future work

# What is an embedded system?

Embedded system



Minicomputer designed  
to satisfy specific  
requirements

# Embedded Systems

## Daily activities



## Embedded systems



## Industry



# Design challenge



Technology offers more  
and more functionalities



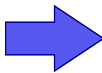
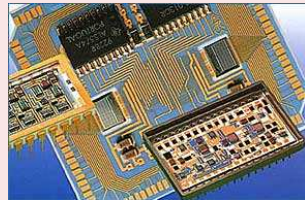
## Design challenge



Technology offers more and more functionalities



Design of embedded systems becomes more and more complex

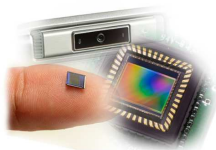


# Designer objectives

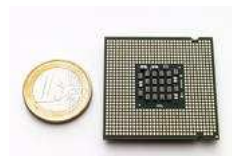
Designers must take into account:



Power  
consumption (W)



Area (mm<sup>2</sup>)



Cost (\$)



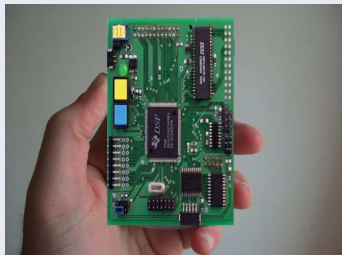
# Architecture

## Embedded system



# Architecture

## Embedded system

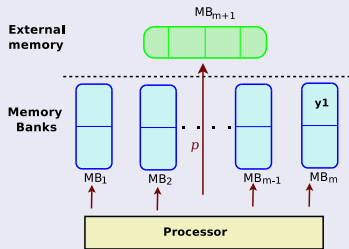


# Architecture

Embedded system



Simplified architecture



# Application



## Functionalities

- phoning
- texting
- digital camera
- web browsing
- etc...

# Application

## Embedded system



## Functionalities

- phoning
- texting
- digital camera
- web browsing
- etc...

## C source code

```
1 /* LMS dual-channel filter */
2
3 void main() {
4     int y11, y12, y21, y22, e1, e2;
5     for(int k=0;k<10;k++)
6     {
7         int n = (k+10)%L;
8         y11=0;
9         for(int i=0;i<10;i++)
10            { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
11        y12=0;
12        for(int i=0;i<10;i++)
13            { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
14        e1 = y1[n]-y11-y12; /* error */
15        H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
16        H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
17        y21=0;
18        for(int i=0;i<10;i++)
19            { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
20        y22=0;
21        for(int i=0;i<10;i++)
22            { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
23        e2 = y2[n]-y21-y22;
24        H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
25        H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
```

# Data structures

## Application

### Source code

```
1 /* LMS dual-channel filter */
2
3 void main() {
4     int y11, y12, y21, y22, e1, e2;
5     for(int k=0;k<10;k++)
6     {
7         int n = (k+10)%L;
8         y11=0;
9         for(int i=0;i<10;i++)
10        { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
11        y12=0;
12        for(int i=0;i<10;i++)
13        { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
14        e1 = y1[n]-y11-y12; /* error */
15        H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
16        H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
17        y21=0;
18        for(int i=0;i<10;i++)
19        { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
20        y22=0;
21        for(int i=0;i<10;i++)
22        { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
23        e2 = y2[n]-y21-y22;
24        H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
25        H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
```



# Data structures

## Application

### Source code

```
1 /* LMS dual-channel filter */
2
3 void main() {
4     int n = (k+10)%L;
5     y11=0;
6     for(int i=0;i<10;i++)
7     { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
8     y12=0;
9     for(int i=0;i<10;i++)
10    { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
11    e1 = y1[n]-y11-y12; /* error */
12    H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
13    H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
14    y21=0;
15    for(int i=0;i<10;i++)
16    { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
17    y22=0;
18    for(int i=0;i<10;i++)
19    { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
20    e2 = y2[n]-y21-y22;
21    H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
22    H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
```



# Data structures

## Application

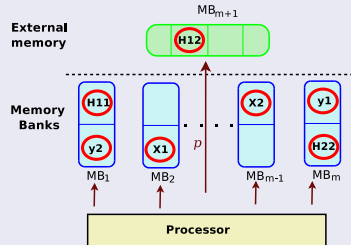
### Source code

```

1 /* LMS dual-channel filter */
2
3 void main() {
4     int y11, y12, y21, y22, e1, e2;
5     for(int k=0;k<10;k++)
6     {
7         int n = (k+10)%L;
8         y11=0;
9         for(int i=0;i<10;i++)
10        { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
11        y12=0;
12        for(int i=0;i<10;i++)
13        { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
14        e1 = y1[n]-y11-y12; /* error */
15        H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
16        H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
17        y21=0;
18        for(int i=0;i<10;i++)
19        { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
20        y22=0;
21        for(int i=0;i<10;i++)
22        { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
23        e2 = y2[n]-y21-y22;
24        H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
25        H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
    
```



## Memory architecture





# Data structures

## Application

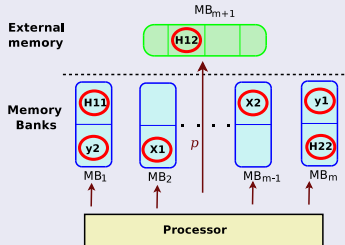
### Source code

```

1 /* LMS dual-channel filter */
2
3 void main() {
4     int y11, y12, y21, y22, e1, e2;
5     for(int k=0;k<10;k++)
6     {
7         int n = (k+10)%L;
8         y11=0;
9         for(int i=0;i<10;i++)
10        { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
11        y12=0;
12        for(int i=0;i<10;i++)
13        { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
14        e1 = y1[i]-y11-y12; /* error */
15        H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
16        H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
17        y21=0;
18        for(int i=0;i<10;i++)
19        { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
20        y22=0;
21        for(int i=0;i<10;i++)
22        { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
23        e2 = y2[n]-y21-y22;
24        H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
25        H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
    
```



## Memory architecture



Memory allocation impacts power consumption, area and cost

# Processor

## Application

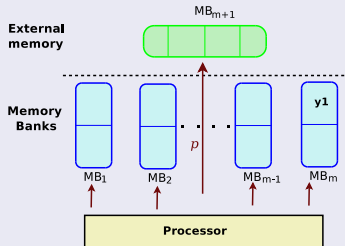
### Source code

```

1 /* LMS dual-channel filter */
2
3 void main() {
4     int n = (k+10)%L;
5     y11=0;
6     for(int i=0;i<10;i++)
7     { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L] - y11; }
8     y12=0;
9     for(int i=0;i<10;i++)
10    { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
11    e1 = y1[n]-y11-y12; /* error */
12    H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
13    H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
14    y21=0;
15    for(int i=0;i<10;i++)
16    { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
17    y22=0;
18    for(int i=0;i<10;i++)
19    { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
20    e2 = y2[n]-y21-y22;
21    H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
22    H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }

```

## Memory architecture



# Processor

## Application

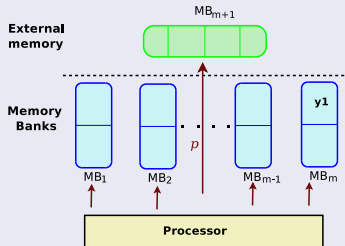
### Source code

```

1 /* LMS dual-channel filter */
2
3 void main() {
4     int n = (k+10)%L;
5     y11=0;
6     for(int i=0;i<10;i++)
7     {
8         int n = (k+10)%L;
9         y11=0;
10        for(int i=0;i<10;i++)
11        {
12            X1 * H11 y11; }
13        y12=0;
14        for(int i=0;i<10;i++)
15        {
16            y12 = X2[(i+k)%L]*H12[((L-1+k-i)%L)+y12];
17            e1 = y1[n]-y11-y12; /* error */
18            H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
19            H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
20            y21=0;
21            for(int i=0;i<10;i++)
22            {
23                y21 = X1[(i+k)%L]*H21[((L-1+k-i)%L)+y21];
24                y22=0;
25                for(int i=0;i<10;i++)
26                {
27                    y22 = X2[(i+k)%L]*H22[((L-1+k-i)%L)+y22];
28                    e2 = y2[n]-y21-y22;
29                    H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
30                    H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
31
32

```

## Memory architecture



# Processor

## Application

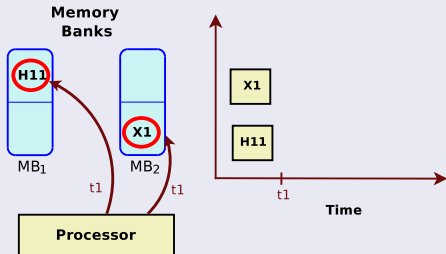
### Source code

```

1 /* LMS dual-channel filter */
11 void main() {
12   int y11, y12, y21, y22, e1, e2;
13   for(int k=0;k<10;k++)
14   {
15     int n = (k+10)%L;
16     y11=0;
17     for(int i=0;i<10;i++)
18     { y11 = X1 * H11 y11; }
19     y12=0;
20     for(int i=0;i<10;i++)
21     { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
22     e1 = y1[n]-y11-y12; /* error */
23     H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
24     H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
25     y21=0;
26     for(int i=0;i<10;i++)
27     { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
28     y22=0;
29     for(int i=0;i<10;i++)
30     { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
31     e2 = y2[n]-y21-y22;
32     H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
33     H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
  
```



## Loading process



# Processor

## Application

### Source code

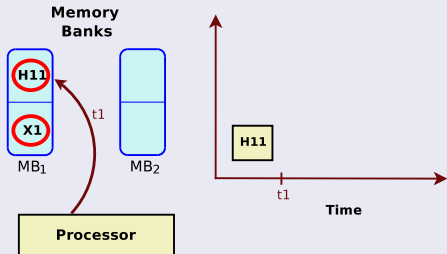
```

1 /* LMS dual-channel filter */
2
3 void main() {
4     int y11, y12, y21, y22, e1, e2;
5     for(int k=0;k<10;k++)
6     {
7         int n = (k+10)%L;
8         y11=0;
9         for(int i=0;i<10;i++)
10        { y11 = X1 * H11 y11; }
11        y12=0;
12        for(int i=0;i<10;i++)
13        { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
14        e1 = y1[n]-y11-y12; /* error */
15        H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
16        H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
17        y21=0;
18        for(int i=0;i<10;i++)
19        { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
20        y22=0;
21        for(int i=0;i<10;i++)
22        { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
23        e2 = y2[n]-y21-y22;
24        H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
25        H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }

```



## Loading process



# Processor

## Application

### Source code

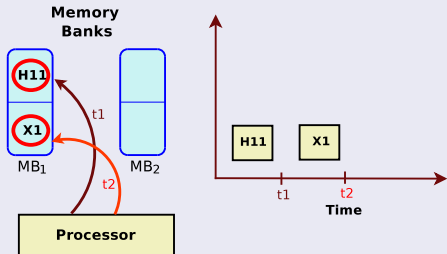
```

1 /* LMS dual-channel filter */
2
3 void main() {
4     int y11, y12, y21, y22, e1, e2;
5     for(int k=0;k<10;k++)
6     {
7         int n = (k+10)%L;
8         y11=0;
9         for(int i=0;i<10;i++)
10        { y11 = X1 * H11 y11; }
11        y12=0;
12        for(int i=0;i<10;i++)
13        { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
14        e1 = y1[n]-y11-y12; /* error */
15        H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
16        H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
17        y21=0;
18        for(int i=0;i<10;i++)
19        { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
20        y22=0;
21        for(int i=0;i<10;i++)
22        { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
23        e2 = y2[n]-y21-y22;
24        H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
25        H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }

```



## Loading process



# Conflicts

## Application

### Source code

```
1 /* LMS dual-channel filter */
2
3 void main() {
4     int n = (k+10)%L;
5     y11=0;
6     for(int i=0;i<10;i++)
7     {
8         int n = (k+10)%L;
9         y11=0;
10        for(int i=0;i<10;i++)
11        {
12            y11 = X1 * H11 y11; }
13        y12=0;
14        for(int i=0;i<10;i++)
15        {
16            y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
17        e1 = y1[n]-y11-y12; /* error */
18        H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
19        H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
20        y21=0;
21        for(int i=0;i<10;i++)
22        {
23            y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
24        y22=0;
25        for(int i=0;i<10;i++)
26        {
27            y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
28        e2 = y2[n]-y21-y22;
29        H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
30        H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
```



## Conflict

- Two structures are said to be conflicting if they are required at the same time (e.g.  $X1 * H11$ )

# Conflicts

## Application

### Source code

```
1 /* LMS dual-channel filter */
11 void main() {
12   int y11, y12, y21, y22, e1, e2;
13   for(int k=0; k<10; k++)
14   {
15     int n = (k+10)%L;
16     y11=0;
17     for(int i=0; i<10; i++)
18     {
19       y11 = X1[(i+k)%L] * H11[i+n];
20       y12=0;
21       for(int i=0; i<10; i++)
22       {
23         y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12;
24       }
25       e1 = y1[n]-y11-y12; /* error */
26       H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
27       H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
28       y21=0;
29       for(int i=0; i<10; i++)
30       {
31         y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21;
32         y22=0;
33         for(int i=0; i<10; i++)
34         {
35           y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22;
36         }
37         e2 = y2[n]-y21-y22;
38         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
39         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2;
40     }
41   }
42 }
```



## Cost conflict

- Conflicts have a cost in milliseconds (ms)  
(e.g.  $X1 * H11 \rightarrow$  cost of 100 ms)
- Automatically computed by *SoftExplorer*  
<http://www.softexplorer.fr>



# Conflicts

## Application

### Source code

```

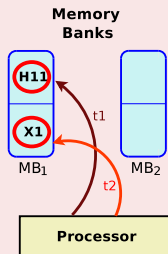
1 /* LMS dual-channel filter */
2
3 void main() {
4     int y11, y12, y21, y22, e1, e2;
5     for(int k=0; k<10; k++)
6     {
7         int n = (k+10)%L;
8         y11=0;
9         for(int i=0; i<10; i++)
10        { y11 = X1[(i+n)%L] * H11[i]; }
11        y12=0;
12        for(int i=0; i<10; i++)
13        { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
14        e1 = y1[n]-y11-y12; /* error */
15        H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
16        H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
17        y21=0;
18        for(int i=0; i<10; i++)
19        { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
20        y22=0;
21        for(int i=0; i<10; i++)
22        { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
23        e2 = y2[n]-y21-y22;
24        H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
25        H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }

```



## Open conflict

X1 and H11 are allocated in the same memory bank → cost is paid



# Conflicts

## Application

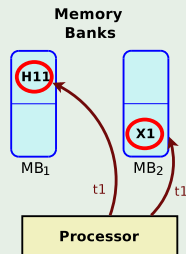
### Source code

```
1 /* LMS dual-channel filter */
11 void main() {
12   int y11, y12, y21, y22, e1, e2;
13   for(int k=0; k<10; k++)
14   {
15     int n = (k+10)%L;
16     y11=0;
17     for(int i=0; i<10; i++)
18     {
19       y11 = X1[(i+n)%L] * H11[i];
20       y12=0;
21       for(int i=0; i<10; i++)
22       {
23         y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12;
24       }
25       e1 = y1[n]-y11-y12; /* error */
26       H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
27       H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
28       y21=0;
29       for(int i=0; i<10; i++)
30       {
31         y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21;
32       }
33       y22=0;
34       for(int i=0; i<10; i++)
35       {
36         y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22;
37       }
38       e2 = y2[n]-y21-y22;
39       H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
40       H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2;
41     }
42   }
43 }
```



## Closed conflict

X1 and H11 are assigned to different memory banks  
→ no cost



# Conflicts

## Application

### Source code

```
1 /* LMS dual-channel filter */
11 void main() {
12   int y11, y12, y21, y22, e1, e2;
13   for(int k=0;k<10;k++)
14   {
15     int n = (k+10)%L;
16     y11=0;
17     for(int i=0;i<10;i++)
18     { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
19     y12=0;
20     for(int i=0;i<10;i++)
21     { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
22     e1 = y1[n]-y11-y12; /* error */
23     H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
24     H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
25     y21=0;
26     for(int i=0;i<10;i++)
27     { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
28     y22=0;
29     for(int i=0;i<10;i++)
30     { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
31     H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
32     H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
```



## Special cases

- *Auto-conflict*: a data structure is in conflict with itself (always open)  
For example  $H21[n+1] = H21[n]$
- Isolated data structure: a data structure is not in conflict with any other one

# Conflict Graph $G = (X, U)$

## Application

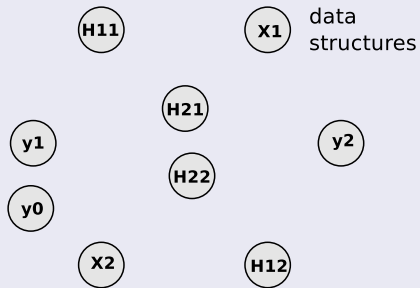
### Source code

```

1 /* LMS dual-channel filter */
11 void main() {
12   int y11, y12, y21, y22, e1, e2;
13   for(int k=0;k<10;k++)
14   {
15     int n = (k+10)%L;
16     y11=0;
17     for(int i=0;i<10;i++)
18     { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
19     y12=0;
20     for(int i=0;i<10;i++)
21     { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
22     e1 = y1[n]-y11-y12; /* error */
23     H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
24     H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
25     y21=0;
26     for(int i=0;i<10;i++)
27     { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
28     y22=0;
29     for(int i=0;i<10;i++)
30     { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
31     e2 = y2[n]-y21-y22;
32     H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
33     H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
    
```



## Conflict graph



# Conflict Graph $G = (X, U)$

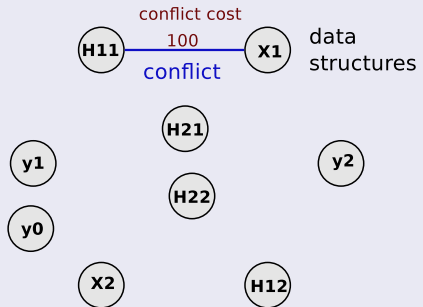
## Application

### Source code

```
1 /* LMS dual-channel filter */
2
3 void main() {
4     int y11, y12, y21, y22, e1, e2;
5     for(int k=0;k<10;k++)
6     {
7         int n = (k+10)%L;
8         y11=0;
9         for(int i=0; i<10;i++)
10        { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]; y11; }
11        y12=0;
12        for(int i=0;i<10;i++)
13        { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
14        e1 = y1[n]-y11-y12; /* error */
15        H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
16        H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
17        y21=0;
18        for(int i=0;i<10;i++)
19        { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
20        y22=0;
21        for(int i=0;i<10;i++)
22        { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
23        e2 = y2[n]-y21-y22;
24        H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
25        H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
```



## Conflict graph



# Conflict Graph $G = (X, U)$

## Application

### Source code

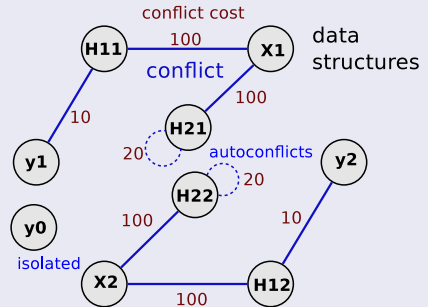
```

1 /* LMS dual-channel filter */
11 void main() {
12   int y11, y12, y21, y22, e1, e2;
13   for(int k=0;k<10;k++)
14     {
15       int n = (k+10)%L;
16       y11=0;
17       for(int i=0; i<10;i++)
18         { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L] - y11; }
19       y12=0;
20       for(int i=0; i<10;i++)
21         { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L] - y12; }
22       e1 = y1[n]-y11-y12; // error
23       H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
24       H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
25       y21=0;
26       for(int i=0; i<10;i++)
27         { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L] - y21; }
28       y22=0;
29       for(int i=0; i<10;i++)
30         { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L] + y22; }
31       e2 = y2[n]-y21-y22;
32       H21[(n+1)%L] = H21[n] - mu21*X1[n]*e2;
33       H22[(n+1)%L] = H22[n] - mu22*X2[n]*e2; } }

```

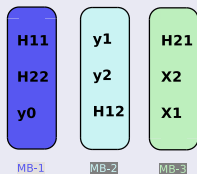


## Conflict graph



## Solution

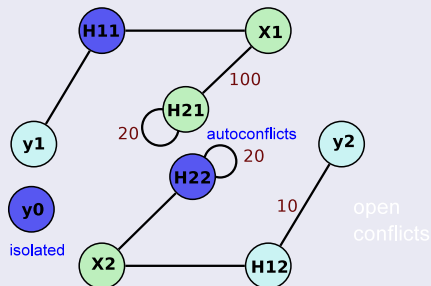
## Memory allocation



Memory banks → colors



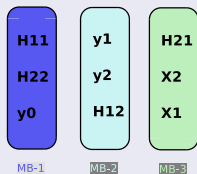
## Coloration



Total cost: 150 ms

## Solution

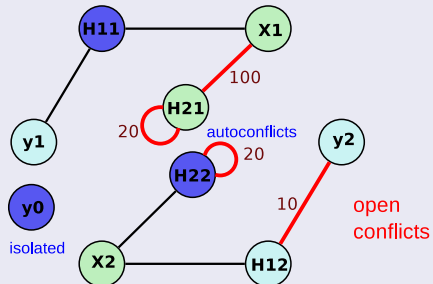
## Memory allocation



Memory banks → colors



## Coloration



Total cost: 150 ms



# Outline

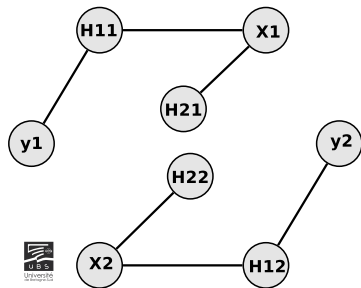
- 1 Introduction
- 2 **Memory allocation problems**
  - Unconstrained memory allocation problem
  - Allocation with constraint on the number of memory banks
  - General memory allocation problem
  - Dynamic memory allocation problem
- 3 Conclusions and future work

# Unconstrained memory allocation problem

- From now, auto-conflicts are ignored because they are always open

## Objective

Find the minimum number of memory banks for which all conflicts are closed



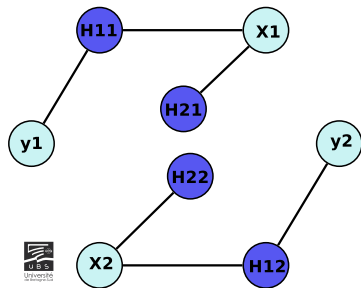
Finding the minimum number of colors to color graph  
→ **Chromatic number**  $\chi$

# Unconstrained memory allocation problem

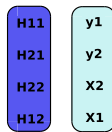
- From now, auto-conflicts are ignored because they are always open

## Objective

Find the minimum number of memory banks for which all conflicts are closed



2 colors  $\rightarrow$  2 memory banks



Memory banks

## New upper bounds

- Easily computable, even for large graphs
  - Outperform the upper bounds from literature
  - But far from the optimal solution
- 
- Sophisticated methods spend too much time
  - Problem solved repeatedly in CAD software
- 
- Upper bounds provide a satisfactory solution

## Allocation with constraint on the number of memory banks

- Fixed number of memory banks
- Cost of conflicts

### Objective

Find memory allocation for data structures such that total cost of open conflicts is minimized

- $k$ -weighted graph coloring problem

Proposed approach from operations research:

→ Memetic Algorithm hybridized with Tabu Search

## Allocation with constraint on the number of memory banks

- Fixed number of memory banks
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→ Memetic Algorithm hybridized with Tabu Search

## Allocation with constraint on the number of memory banks

- Fixed number of memory banks
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- $k$ -weighted graph coloring problem

Proposed approach from operations research:  
→ Memetic Algorithm hybridized with Tabu Search

## Memetic Algorithm

Population

Solutions





## Memetic Algorithm

Population

Solutions



Random  
selection



Cross parents

Parents solutions



## Memetic Algorithm

Population

Solutions



Random  
selection



Cross parents

Parents solutions



Better  
characteristics

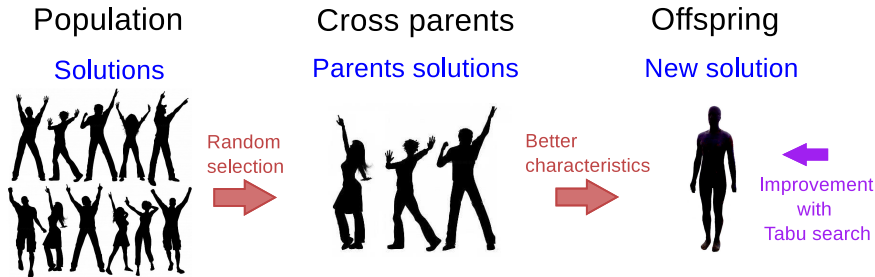


Offspring

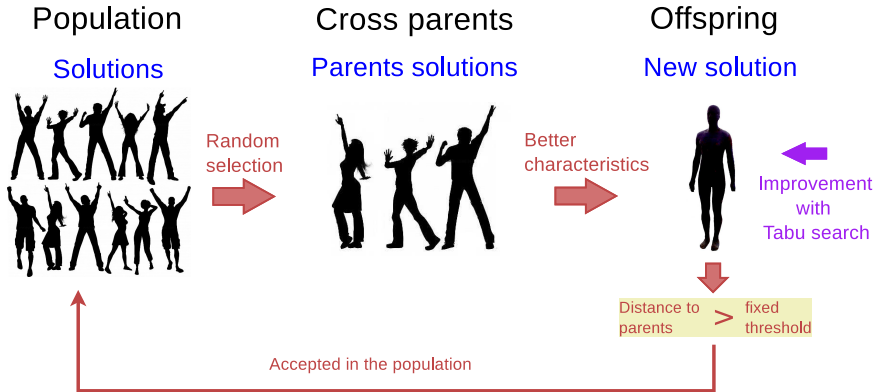
New solution



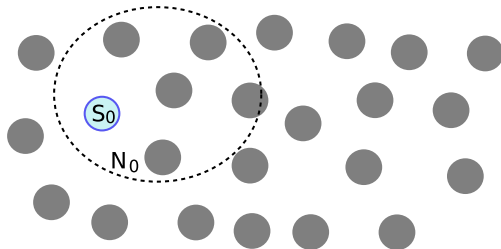
## Memetic Algorithm



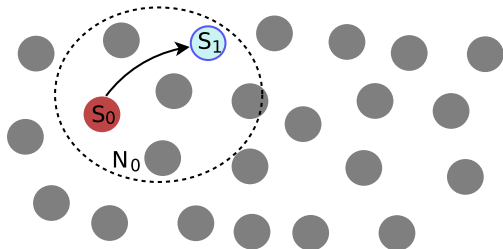
# Memetic Algorithm



## Tabu Search



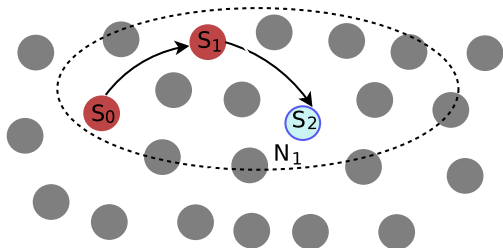
## Tabu Search



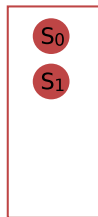
Tabu list



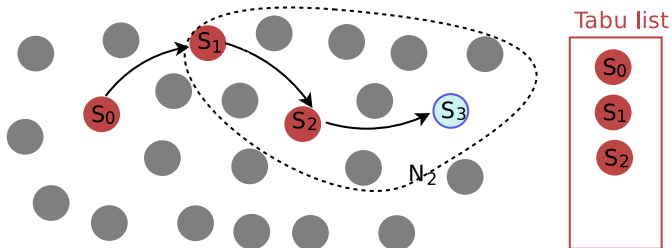
## Tabu Search



Tabu list

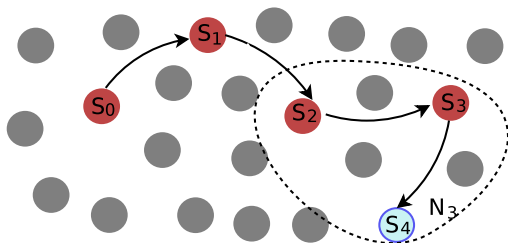


## Tabu Search





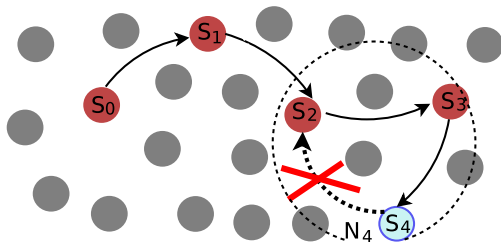
## Tabu Search



Tabu list



## Tabu Search



Tabu list



# General memory allocation problem

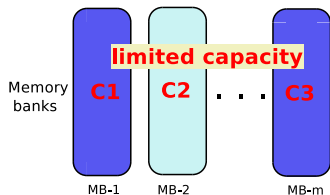
Same characteristics as previous problem, plus

- Sizes of data structures
- Number of accesses of data structures

# General memory allocation problem

Same characteristics as previous problem, plus

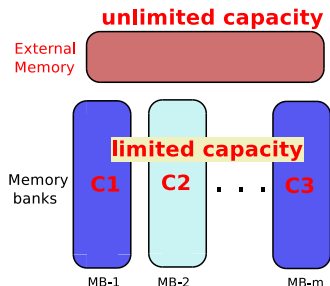
- Sizes of data structures
- Number of accesses of data structures
- Limited capacity of memory banks



# General memory allocation problem

Same characteristics as previous problem, plus

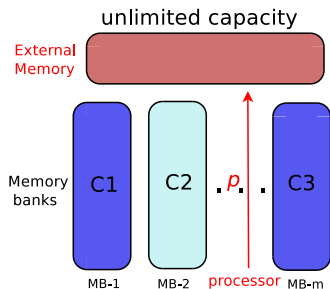
- Sizes of data structures
- Number of accesses of data structures
- Limited capacity of memory banks
- External memory  
→ unlimited capacity



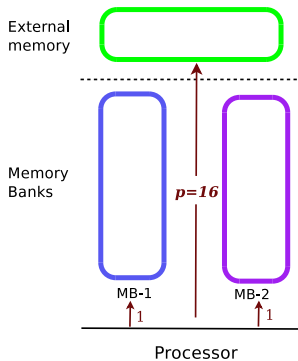
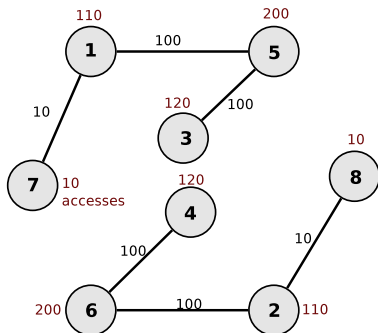
# General memory allocation problem

Same characteristics as previous problem, plus

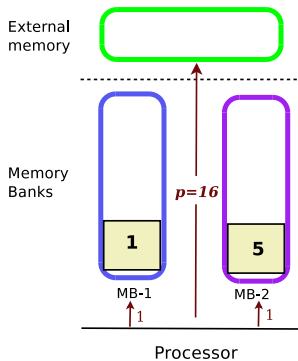
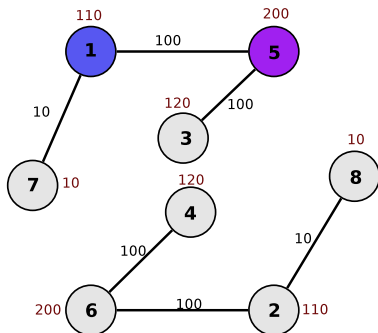
- Sizes of data structures
- Number of accesses of data structures
- Limited capacity of memory banks
- External memory  $\rightarrow$  unlimited capacity
- Access to external memory  $\rightarrow p$



# Example

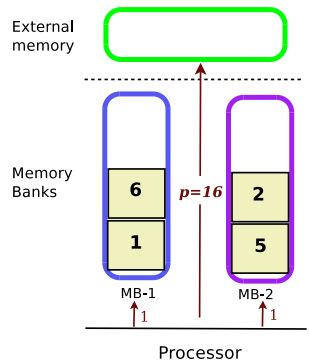
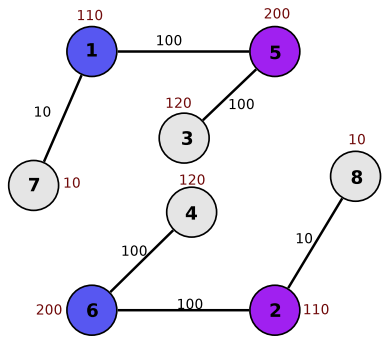


# Example

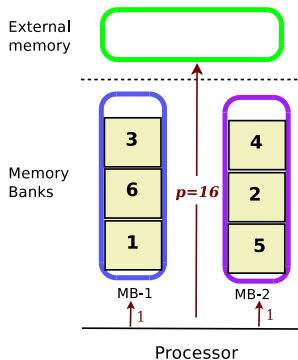
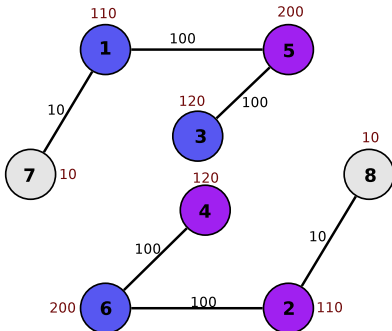




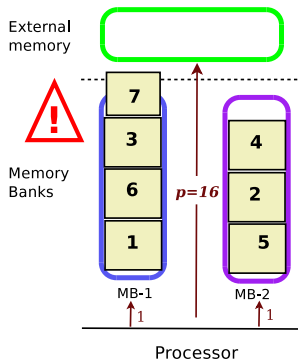
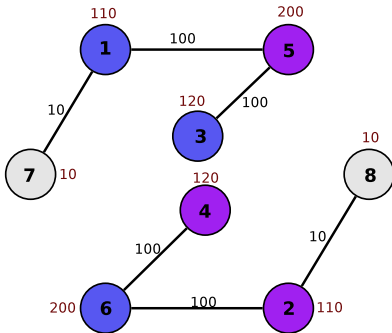
# Example



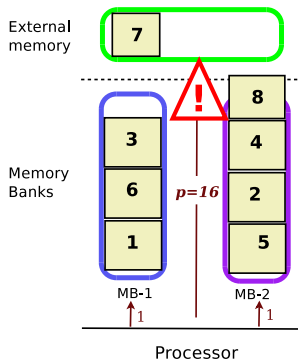
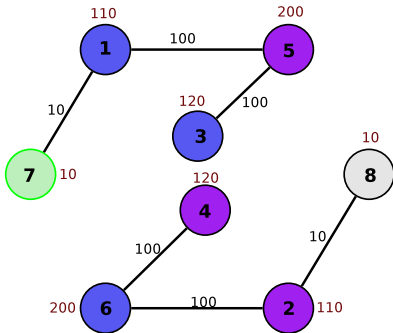
# Example



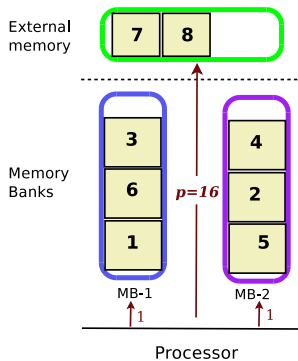
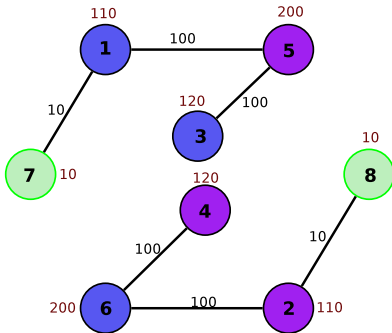
# Example



# Example



# Example



# General memory allocation problem

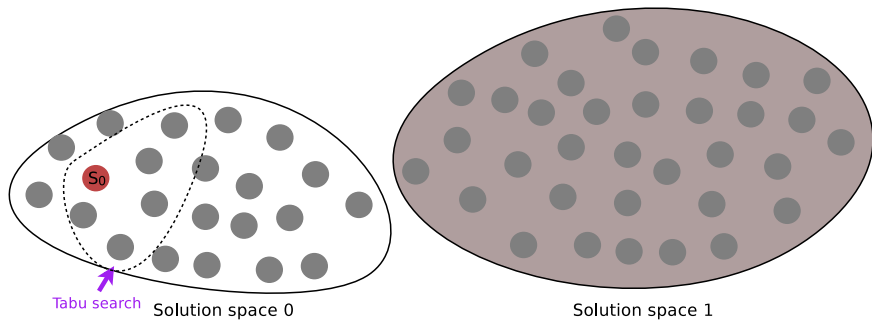
## Objective:

For a given number of capacitated memory banks and an external memory, find a memory allocation for data structures such that the time spent accessing these data is minimized.

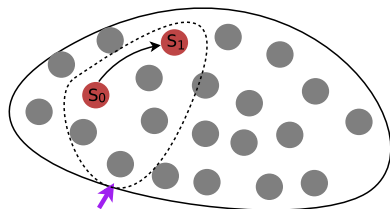
## Proposed approach:

→ Variable Neighborhood Search-Tabu Search (VNS-TS)

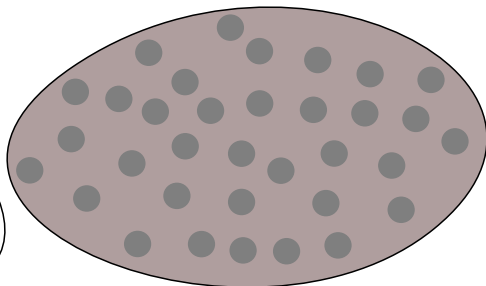
# Hybrid approach (VNS-TS)



# Hybrid approach (VNS-TS)



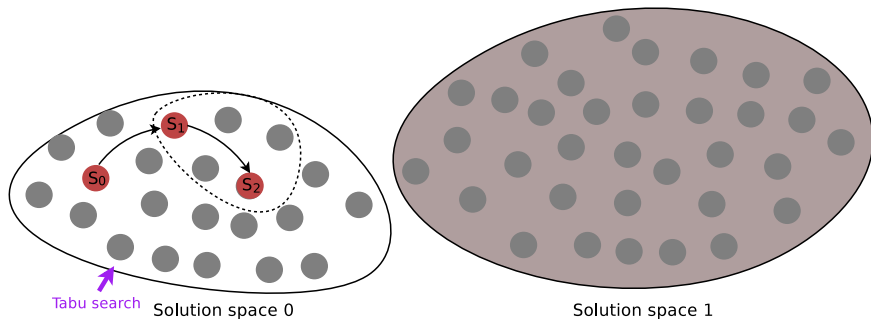
Tabu search Solution space 0



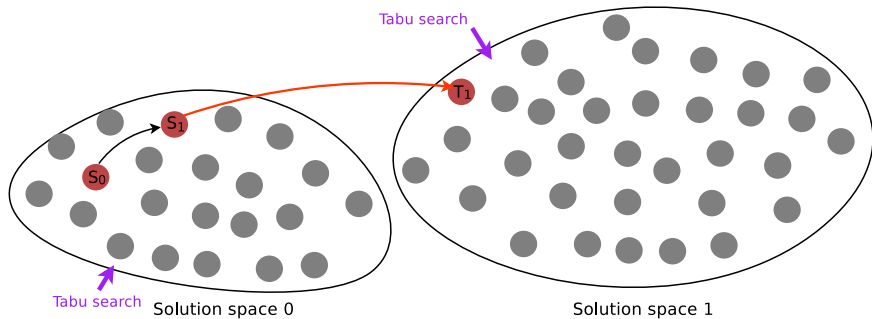
Solution space 1



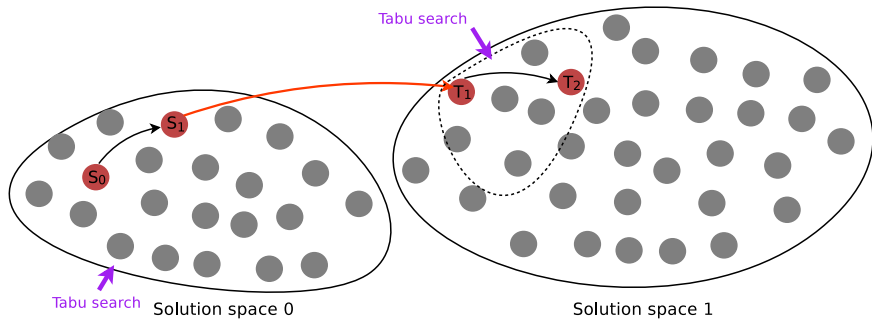
# Hybrid approach (VNS-TS)



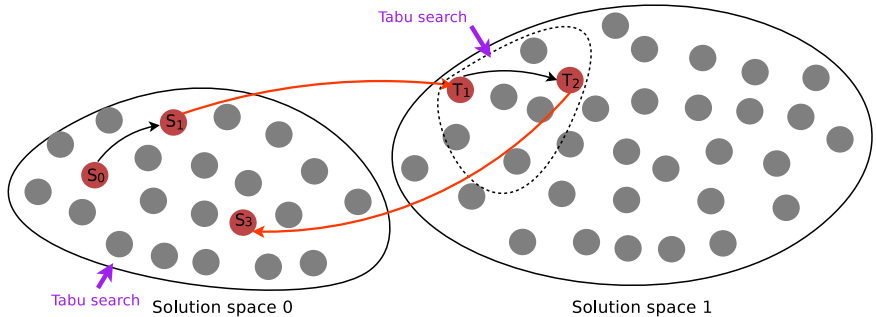
# Hybrid approach (VNS-TS)



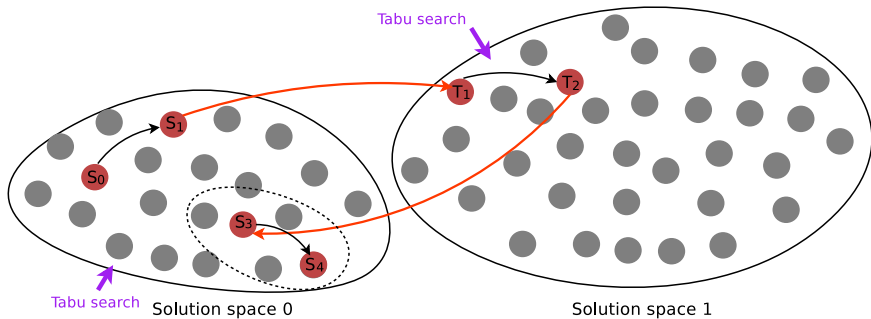
# Hybrid approach (VNS-TS)



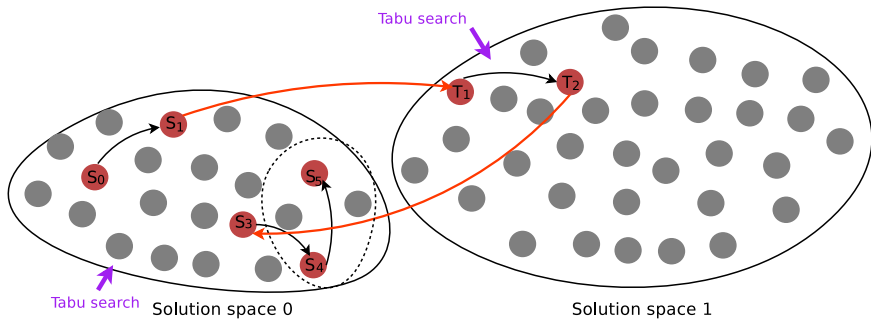
# Hybrid approach (VNS-TS)



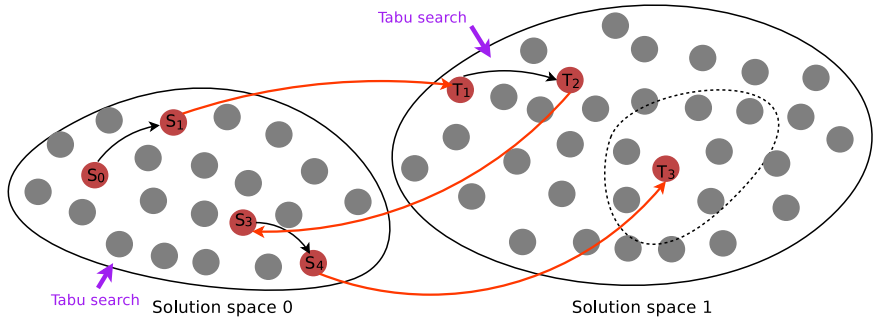
# Hybrid approach (VNS-TS)



# Hybrid approach (VNS-TS)



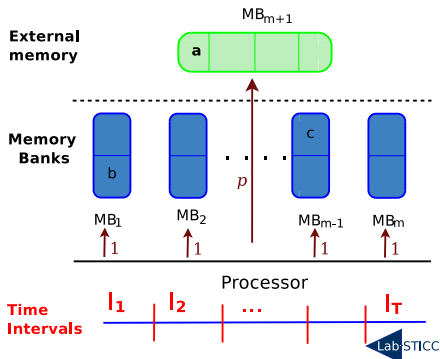
# Hybrid approach (VNS-TS)



# Dynamic memory allocation problem

Same characteristics as general memory allocation problem, plus

- Application time split into time intervals

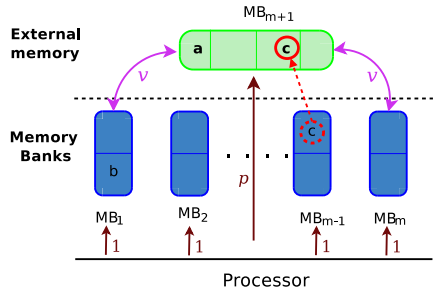




# Dynamic memory allocation problem

Same characteristics as general memory allocation problem, plus

- Application time split into time intervals
- Transfer rates for data structures
  - from external memory to memory banks,  $v$



Transfer time:

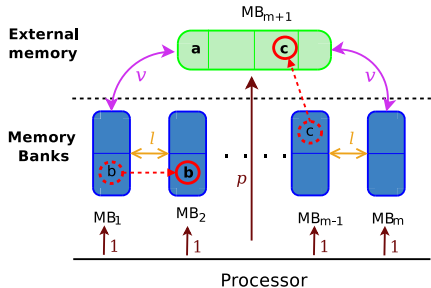
$$C \rightarrow S_c * v \dots$$



# Dynamic memory allocation problem

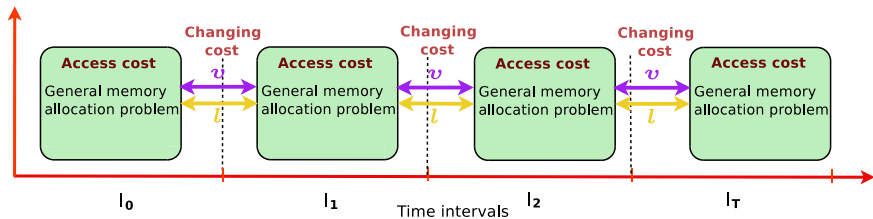
Same characteristics as general memory allocation problem, plus

- Application time split into time intervals
- Transfer rates for data structures
  - from external memory to memory banks,  $v$
  - between memory banks,  $l$



Transfer time:  $b \rightarrow s_b * l$ ,  
 $c \rightarrow s_c * v \dots$

## Dynamic memory allocation problem

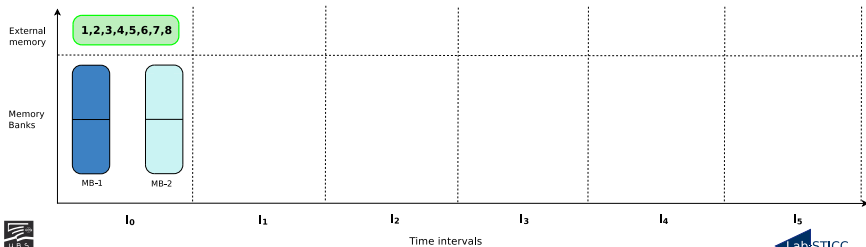


# Example

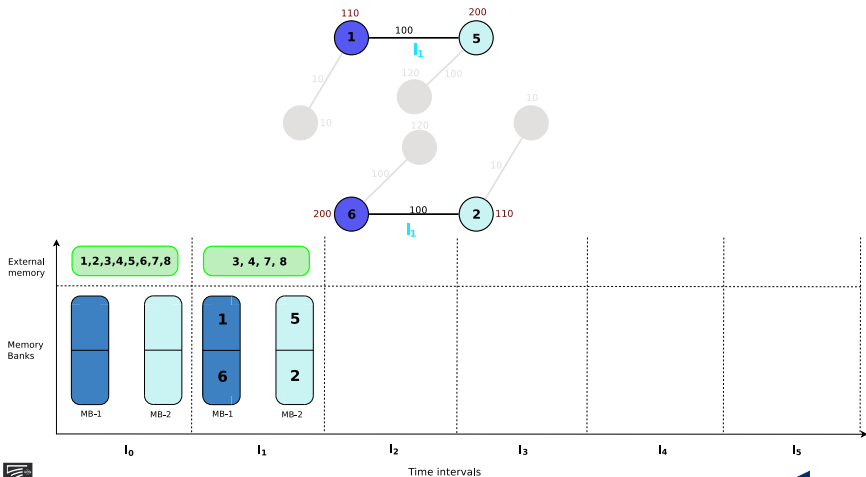
$\rho = 16$  ms, and  $l = v = 1$  ms/ko.

Intervals $t = 1, \dots, 5$	Data structures $\{a_{1,t}, \dots, a_{n_t,t}\}$	Conflicts $(a_{k_1,t}, a_{k_2,t})$	Cost $d_{k,t}$	Access time $e_{a_{i,t},t}$
1	{ 1, 5, 2, 6 }	(1;5)	1,046,529	$e_{1,1} = e_{2,1} =$
		(2;6)	1,046,529	$e_{5,1} = e_{6,1} = 1,046,529$
2	{ 3, 4, 5, 6 }	(3;5)	1,046,529	$e_{3,2} = e_{5,2} =$
		(4;6)	1,046,529	$e_{4,2} = e_{6,2} = 1,046,529$
3	{ 1,5,7 }	(1;7)	1,023	$e_{1,3} = 2,046$
		(1;5)	1,023	$e_{5,3} = e_{7,3} = 1,023$
4	{ 2,6,8 }	(2;6)	1,023	$e_{2,4} = 2,046$
		(2;8)	1,023	$e_{6,4} = e_{8,4} = 1,023$
5	{ 3,4 }	(3;3)	2,046	$e_{3,5} = e_{4,5} = 2,046$
		(4;4)	2,046	

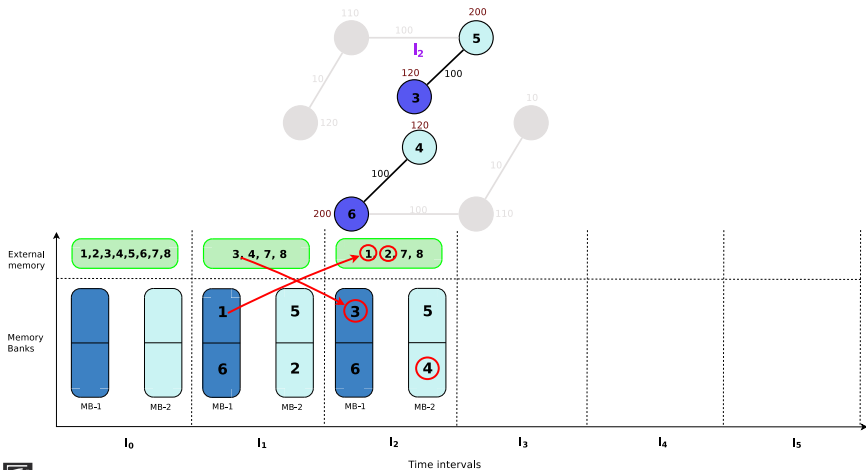
# Example



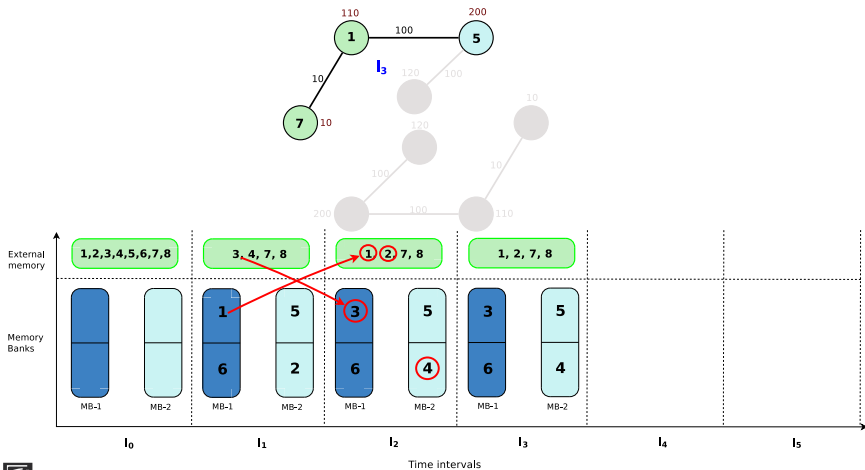
# Example



# Example

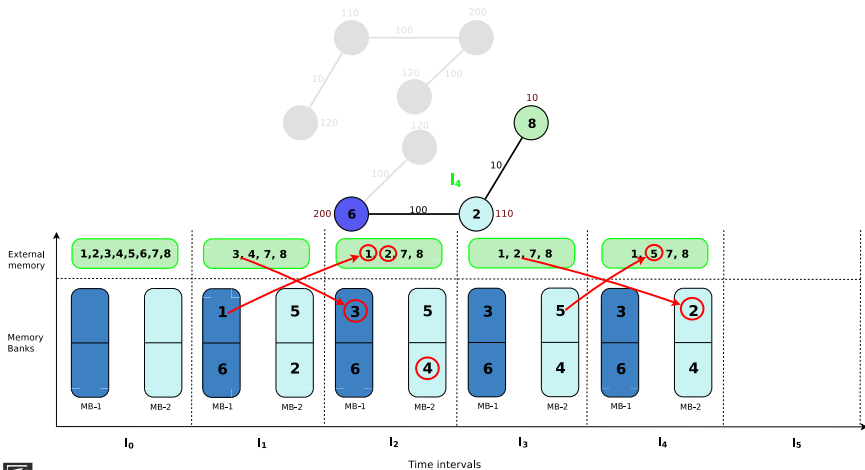


# Example

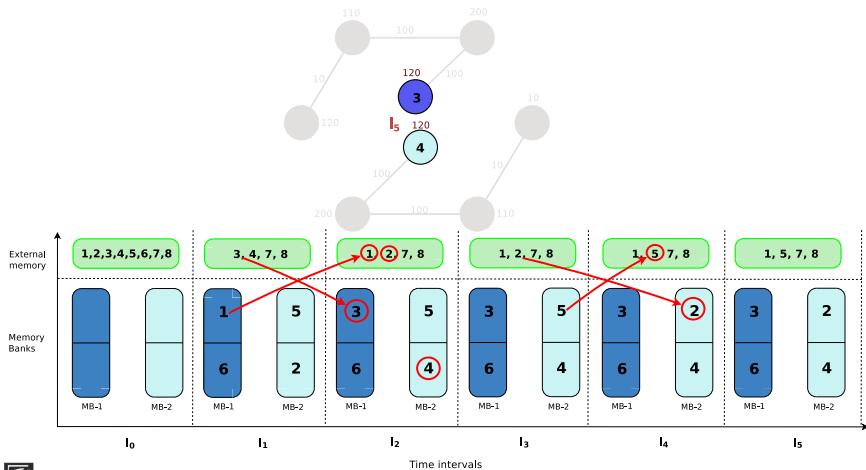




# Example



# Example



# MemExplorer Dynamic

## Objective

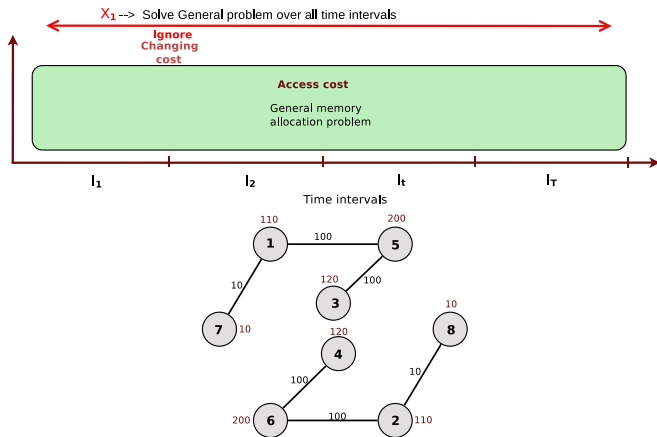
Allocate a memory bank or the external memory to any data structure of the application for **each time interval**

- minimize data access time and data moving time
- satisfy the memory banks' capacity.

Two proposed approaches:

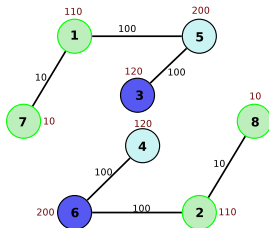
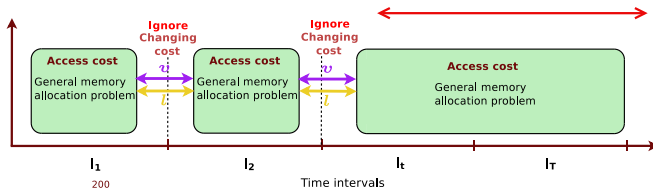
- Long-term approach
- Short-term approach

## Long-term approach

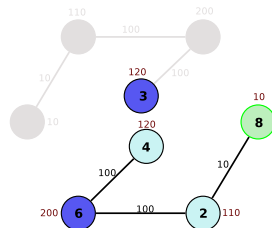


# Long-term approach

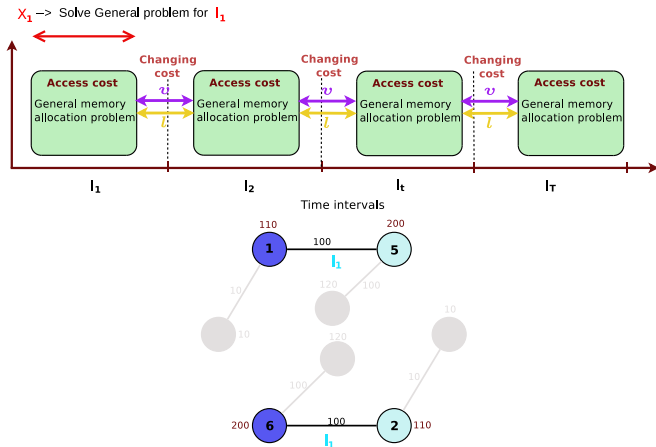
$X_t \rightarrow$  Compare  $X_{t-1}$  and  $M_t$   
 Choose the best one  
 $M_t \rightarrow$  Solve General problem from  $t$  to  $T$



Compare two solutions

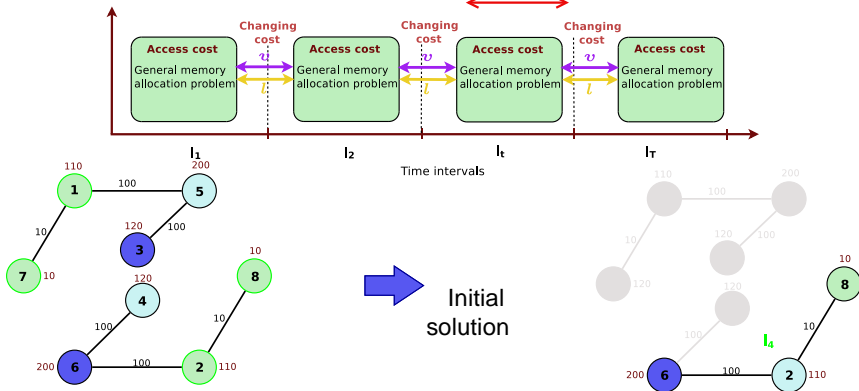


## Short-term approach



## Short-term approach

$X_t \rightarrow$  Solve General problem for current time interval  $I_t$



# Outline

- 1 Introduction
- 2 Memory allocation problems
- 3 Conclusions and future work



# Conclusions

- Four problems to memory allocation
- Approaches inspired from graph coloring algorithms
- Well balanced search in terms of intensification and diversification
- Results are better than ILP (Xpress-MP) and LocalSearch 1.0 (Bouygues e-lab. <http://e-lab.bouygues.com>)
- General memory allocation problem implemented in *SoftExplorer*

## Future work

- Adapt our approaches for the memory allocations with small granularity
  - Extend our approaches for other memory allocation problems
- 
- New upper bounds exploiting graph topology
  - Adapt algorithms for the Bin Packing to memory allocation problems
  - Use the idea of the Knapsack algorithm for our problems
  - Design matheuristics to memory allocation problems

## Future work

- Mid-term approach to combine the benefits of Short term and Long term approaches
- Global approach for the dynamic memory allocation
- Continue the implementation in *SoftExplorer*

# Thank you for your attention

## Publications:

- Two journal articles:
  - Discrete Applied Mathematics
  - Journal of Heuristics
- Three international conferences:  
Evocop, EU/Meeting and CTW09
- Three national conferences:  
Roadef 2011, MajecSTIC and Roadef 2010

